Journal of Research High School

Vol. 2024 (8) 311 – 321
ISSN 2688-3651 [online]

Posted Price Versus Auction Models in Venture Capital Industry

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Received February 1, 2024; Revised June 1, 2024; Accepted June 15, 2024

Abstract

The venture capital industry plays a pivotal role in fostering innovation and driving economic growth by providing required funding to start-ups with long-term growth potential. In this high-risk environment, the choice of pricing mechanisms (whether posted price or auction) for start-up companies can significantly impact investment decisions, allocation of resources, efficiency, and finally success of the ventures. This research paper delves into an in-depth comparison of a first-price sealed-bid auction against a posted price system within the context of the venture capital industry. The goal is to compare their relative advantages, limitations, and overall implications for venture capitalists and entrepreneurs by starting with deriving optimal strategies in a standard posted price and auction model with bid preparation costs. The second part of this paper will focus on maximising the expected utility and the total capital raised by the start-up, while comparing optimal strategies within each model. The analysis shows a negative relationship between bid preparation costs and the total revenue generated, which substantiates the existence of miscoordination due to fear of wasted efforts and predicts the effect of priority on efficiency. However, the extent to which each factor impacts the equilibrium varies greatly with changes in the pricing mechanism.

Keywords: Auction, Posted-price, Game Theory, Pricing mechanism, Economic evaluation, Comparing

1. Introduction

1.1 Background

The venture capital industry is one of the fastest growing markets, with an expected growth rate of 21.75% between 2023 and 2028. A venture capitalist is a private equity investor who provides financial capital to start-ups with high growth potential in exchange for an equity stake typically in an unlisted company, whose shares are not traded in a stock market and where both risk and rewards are high. It is most common for one venture capitalist to consider dozens of potential start-ups for a single investment. In this case, the decision to invest involves the objective evaluation of the potential growth of the company and the optimistic bias of the entrepreneur who may exaggerate their profit projection. However, as the venture capital industry grows, venture capitalists engage in increasing competition in pursuit of the next private start-up they would like to invest in, typically one with a valuation of over one billion dollars.

When venture capitalists engage in competition, one of the key questions for start-ups is how to best raise financial capital by effectively choosing a venture capitalist. There are two main ways for start-ups to choose a venture capitalist: posting a fixed price or running an auction. Posted prices are fixed prices determined by the start-up at which a certain portion of the company is traded. For the sake of simplicity, the possibility of bargaining is being disregarded. The second method is running a first-price sealed-bid auction, where each player of type v_i submits a bid b_i to the auctioneer without revealing its contents to any other bidder. The winner of the auction is the bidder with the highest bid. It is important to note that the primary objective of the venture capitalist firm is to grow the new venture, make it a profitable business, and thereby generate maximum revenue at minimum cost. Therefore, this paper relies on the derivation of the 'maximum' utility in posted price and auction models.

1.2 Significance

Both first-price sealed-bid auctions and posted prices are established strategies for start-ups to attract and decide on venture capitalists. However, given the surge in competition in recent years and the changes in the market that have come with the introduction of new financing platforms (crowdfunding, angel investors), re-evaluating pricing mechanisms has become necessary to maximise revenue and adapt to these rapid changes.

2. Background

2.1 Discussion of Related Literature

Over the years, there has been significant research on the role of venture capitalists, how they have evolved over time, and the decision-making process behind investing in a start-up. For example, a paper by Paul Gompers (2001), an economist, analysed the growth of venture capitalists, its funding cycles, the intrinsic risks involved, and potential exit strategies. While the primary focus of the paper is empirical research in the venture capital industry, it also highlights certain unanswered questions, including those regarding risk and reward comparisons and the impact of dynamic growth in the industry on the companies they fund. The answers to these questions remain uncertain. More recently, Gornall et al. (2021) detailed the step-by-step decision-making process of a venture capitalist, from identifying the set of start-ups to down selecting to potentially hand holding the start-up through the process by providing managerial guidance, till the final exit. According to the findings, financial evaluation was not important in identifying the start-up, rather the potential return at exit was the major deciding factor. Additionally, a deeper analysis of the investment framework of venture capitalists by Corea et al (2021) identified the potential harmful impacts of relying on gut feeling, biases and heuristics to decide on the most profitable investment. They developed a data-driven framework that includes a smart checklist of twenty-one relevant features that may help investors in selecting start-ups with a higher probability of success, including founders' track record, history of debt, social media presence, and many more. The challenges highlighted by the research above are largely a result of the adverse selection, which occurs when there is asymmetric or unequal information between buyers and sellers.

In light of this, many researchers have used Game Theory to analyze start-up and Venture capital relationships. Elitzur and Gavious (2003) developed a multi-period game theoretic model with moral hazard, a situation when an individual has an incentive to increase its exposure to risk. This model is among the most realistic as it is longitudinal and considers that investment is made in stages. However, further exploration could focus on the contracts among VCs who syndicate together. In contrast, Chen (2016) uses a complete information dynamic game to discuss the different types of cooperation that can exist between start-ups and venture investors: property or non-property cooperation. Non-property cooperation is a type of cooperation in which the venture investor provides financial resources to the start-up but does not have any ownership of the company. Property cooperation, on the other hand, is a type of cooperation in which the venture investor provides financial resources to the start-up and also receives an ownership stake in the company. The results of the game analysis indicates that if a start-up is facing a high risk of failure, it may want to choose non-property cooperation, as this will reduce the amount of risk that the venture investor bears. However, if a start-up has a high chance of success, they may want to choose property cooperation, as this will give them access to more capital and resources. The analysis is relatively simple, however, and does not take into account many variables such as the ability of entrepreneurs to succeed in the face of crisis and the amount of trust the relationship between the start-up and the venture capitalist contains. Chen (2016) and Elitzur and Gavious (2003) use starkly different models, which reveal different insights into the workings of the Venture capital market.

As the venture capital industry grows, so does the importance of analysing the effectiveness of different structural models. For instance, Wang (1998) compares auctions and posted price selling in a one-period correlated private-

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values model. The seller of an object must decide whether to sell it by posting a fixed price or by an auction. Without auctioning costs, Wang showed, an auction with reserve pricing is always preferable. With negative auctioning costs, an auction is always preferable. With positive auctioning costs, Wang found that an auction is still preferable when the buyers' valuations is sufficiently dispersed or when the value of the object is sufficiently high. Similarly, Einav et al (2018) drew a comparison between the auctions and the newly introduced posted price sale in online markets. They modelled the choice between auctions and posted prices as a trade-off between competitive price discovery and convenience. They then show that the decline in auctions was not driven by compositional shifts in seller experience or items sold, but by changing seller incentives. They also estimate the demand facing sellers, and document falling sale probabilities and falling relative demand for auctions. Both authors above favour posted prices. Their estimates suggest that the latter is more important for the auction decline. The contrasting evidence, supporting auctions, calls for more research in this field.

Research on game theory in relation to venture capitalists has demonstrated the role of asymmetric information in investment decisions. Yet the role of certain factors remains uncertain. How does the structure influence the extent of inefficiency? What is the role of bid preparation costs in optimal investment strategies? This paper proposes answers to these questions using game theoretical models.

3. Methodology

3.1 Posted Price Investment Model

Consider a situation where there is an innovative start-up company with a breakthrough technology seeking to raise \$1 million for 20% ownership of their company. For the sake of simplicity, we assume two venture capitalists, VC1 and VC2, both of whom recognize the enormous potential of the start-up and want to maximize their returns on investment. However, due to market uncertainties and competitive risks, they must decide whether to invest in the start-up or refrain from investing altogether. Therefore, this game or scenario can be represented as a finite two-player game.

The underlying true value of the asset is denoted as x which is unknown to both VCs. However, given the asymmetric information in the market, the asset's intrinsic value and the 'type' of the competing venture capitalist remains unknown. Therefore, the value-determination process is modeled as follows. Firstly, each venture capitalist receives a private signal about the start-up, based on which they assign a value: v_1 and v_2 respectively, which are uniformly distributed along a function F on [0, s].

Based on their belief about the value of the asset, both players can choose to "invest" or "not invest." The total cost of investment is represented by $c + k$, where c is the amount the venture capitalist invests, and k is the preparation cost. Therefore, irrespective of whether the venture capitalist choses to invest or not, it incurs a cost k which includes the cost of labour, materials, capital, and research. The payoffs for each venture capitalist will depend on their actions and the underlying value x of the asset. Since this is unknown, their payoff equals their estimated value − cost incurred. If VC1 invests and VC2 doesn't, VC1's payoff is $(v_1 - c - k)$. Similarly, if VC2 invests and VC1 doesn't, their payoff is $(v_2 - c - k)$.

In the situation where both invest, a common factor in determining the reward is the size or reputation of the venture capitalist. However, for this simplified model, we assume that their reputations if quantified are equal. Therefore, when both players invest, VC1's payoff is $\left(\frac{v_1}{2} - \frac{c}{2} - k\right)$, and VC2's payoff is $\left(\frac{v_2}{2} - \frac{c}{2} - k\right)$ as the 'preparation costs or costs of preparing the investment remain the same irrespective of the strategies played. Since preparing a losing bid is expensive, each player will invest or even consider the start-up as an investment option if and only if their estimated value is greater than the total cost of investment.

Consider the game to be a simultaneous move game with imperfect or incomplete information, where none of the players are aware of the quality of the asset and the true 'type' of their competing venture capital. Therefore, to understand the equilibrium in this game, a Bayesian Nash equilibrium can be found as it relies on both, the players'

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beliefs and the strategies played. The probability Of VC2 choosing to invest is P_2. The probability of VC2 choosing not to invest is 1-P_2.

Therefore, VC1 will invest if and only if:

$$P_2\left(\frac{v_1}{2}-\frac{c}{2}-k\right)+(1-P_2)(v_1-c-k)\geq 0$$
$$\Rightarrow \frac{P_2v_1}{2}-\frac{cP_2}{2}-kP_2+v_1-v_1P_2-c+cP_2-kP_2\geq 0$$
$$\Rightarrow v_1\left(\frac{P_2}{2}+1-P_2\right)-\frac{cP_2}{2}-c+cP_2-k\geq\ 0$$
$$\Rightarrow v_1\geq c+\frac{2k}{(-P_2+2)}$$

Given that the probability must be consistent with VC2's actual behaviour, there is a cut-off value: $v_1^*=c+\frac{2k}{(-P_2+2)}$ in which case VC2 invests only if $v_2\geq v_2*$ and VC1 invests only if,

$$v_1\geq v_1^*$$

Let P_1 be the probability of VC1 investing.

Since $v_1^*=v_2^*$,

$$P_2=P_1=1-F(v^*)=1-\frac{v^*}{s}$$
$$\Rightarrow v^*=c+\frac{2k}{\left(1-\frac{v^*}{s}+2\right)}$$
$$\Rightarrow v^*-c=\frac{2ks}{(v^*+s)}$$
$$\Rightarrow (v^*)^2+v^*(-c+s)-sc-2ks=0$$
$$v^*=\frac{-(-c+s)+\sqrt{(-c+s)^2-4(1)(-sc-2ks)}}{2}=\frac{c-s+\sqrt{c^2+s^2+2sc+8ks}}{2}$$
$$v_1^*=\frac{c-s+\sqrt{(c+s)^2+8ks}}{2}$$

Therefore, according to the symmetric Bayesian Nash Equilibrium, each venture capitalist must enter if and only if $v_1\geq\frac{c-s+\sqrt{(c+s)^2+8ks}}{2}$

As a result, we can conclude that v_1 is strictly greater than *c*+*k,* and increases with a rise in the bid cost, k. If $k=0$, VC1 would invest when and if $v_1\geq c$. Moreover, if both VC1 and VC2 have values less than $c+k$, none of them invest. When the value lies between $c+k$ and v^*, at least one venture capitalist assigns a value greater than the cost $c+k$, in which case it is expected that the venture capitalists would invest to maximize allocative efficiency. However, according to the model, the start-up does not get invested in, resulting in economic and efficiency loss in the market due to fear of miscoordination amongst the players. This stems from the possibility of both venture capitalists investing, thereby reducing each of their payoffs, or the risk of making a loss given that their value is only marginally higher than $c+k$.

Total Revenue Generated

In the posted price model, the winning venture capitalist simply pays the cost *c* determined by the start-up. Therefore, the revenue generated by the start-up is c, on the condition that the venture capitalist enters the market. This can be represented as:

$$R_p(c)=c[P_1P_2+P_1(1-P_2)+P_2(1-P_1)]$$
$$\Rightarrow R_p(c)=c\left[\left(1-\frac{v^*}{s}\right)^2+2\left(1-\frac{v^*}{s}\right)\left(\frac{v^*}{s}\right)\right]$$

For the sake of notational simplicity, we assume $s = 1$ and $k = 0.5$

$$R_p(c) = c[(1 - v^*)^2 + 2(1 - v^*)(v^*)]$$

$$\Rightarrow c\left[\left(1 - \frac{c-1+\sqrt{(c+1)^2+4}}{2}\right)^2 + 2\left(1 - \frac{c-1+\sqrt{(c+1)^2+4}}{2}\right)\left(\frac{c-1+\sqrt{(c+1)^2+4}}{2}\right)\right]$$

This simplifies to:

$$R_p(c) = c\left[1 - \left(\frac{c-1+\sqrt{(c+1)^2+4}}{2}\right)^2\right]$$

Now, to calculate the optimal value c, we want to maximize the expected payoff given k. Therefore, we must optimise $R_p(c)$ as shown by the following steps:

$$\frac{\mathrm{d}(\mathrm{R_p})}{\mathrm{dx}} = -\frac{1}{2}\left(\frac{3c^3 + 3c^2 + 3c^2\sqrt{c^2+2c+5} + 7c - 5}{\sqrt{c^2+2c+5}} + 1\right)$$

Equating this to zero, the revenue is maximized at $c^* = 0.266$ when $k = 0.5$ as shown in the diagram below.

We notice that the revenue function is concave down by nature due to the trade-off between price and quantity demanded. As c approaches zero, R_p simultaneously decreases until it reaches zero. On the other hand, when the price c is too expensive, revenue decreases, resulting in fewer venture capitalists willing to invest. Therefore, revenue is maximized at some value of c between 1 and $s - k$.

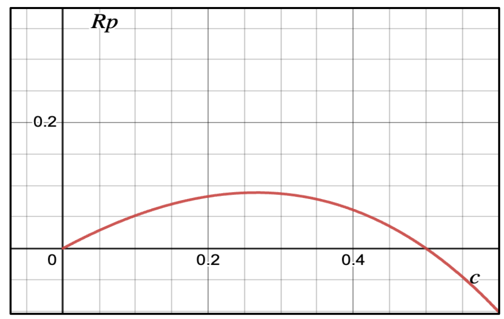

Figure 1: Revenue in the posted price model

Calculating the Probability of Miscoordination Amongst the Players

Geometrically speaking, the area required is the area greater than $(c + k)$ until v^*, which can be expressed as:

$$M_p = 2\big[\big(v^* - (c+k)\big) \times (c+k)\big] + [v^* - (c+k)]^2$$

This simplifies to,

$$M_p = -(c+k)^2 + (v^*)^2$$

Substituting v*,

$$M_p = -(c+k)^2 + (v^*)^2$$

$$\Rightarrow M_p = -(c+k)^2 + \left(\frac{c-s+\sqrt{(c+s)^2+8ks}}{2}\right)^2$$

Consider a fixed value $0 < c < 1$, based on which we plot a graph k against the probability of inefficiency, E. We realise that while E is at a minimum when k is 0 and $1 - c$, it is maximum at intermediate values of k. At $k = 0$, both venture capitalists (VCs) are likely to invest given that there is no cost of bid preparation. At $k = 1$, the VCs cannot have values high enough to overcome the cost since v^* must be <1 by construction. Therefore, it is when the cost of preparation is intermediate that VCs have values greater than $c + k$, but choose not to invest due to the fear of wasted efforts. Here is a plot of k against E where $c = c^* = 0.266$ and $s = 1$, where miscoordination is at a maximum at $k = 0.3987$.

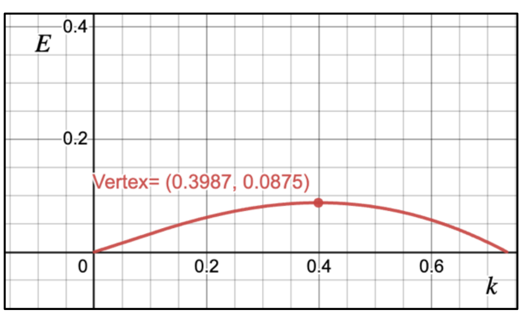

Figure 2: Miscoordination in the posted price model

Therefore, as shown in the diagram, when bid preparation costs increase, inefficiency initially increases and eventually falls.

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Role of Priority in a Posted Price Model

In most investments, the role of venture capitalists is not limited to financing the start-up. Rather, venture capitalists are often expected to bring technical and managerial expertise to help them succeed. Therefore, as Zhu (2019) demonstrated, a well-reputed, established venture capitalist is more likely to attract higher rewards from start-ups. We represent the additional reward based on size and reputation of the venture capitalist using α. The greater the α value, higher is the probability of the established venture capitalist choosing to invest and lower is the efficiency loss of not investing.

We assume that when both venture capitalists invest, their cost and value is increased or decreased by a constant value α that represents the start-up's valuation of the venture capitalist's reputation and expertise. VC1 and VC2's expected payoffs conditional on both investing are now $\alpha(v_2 - c) - k$ and $(1-\alpha)(v_2 - c) - k$. Since the players' corresponding rewards differ based on their reputation, we solve for the asymmetric Nash equilibria. The probability of VC1 investing is P_1, while the probability of VC2 investing is P_2. The condition for VC1 to invest is represented as follows.

$$P_2[\alpha(v_1 - c) - k] + (1 - P_2)[v_1 - c - k] \geq 0$$

Simplifying this we get:

$$\Rightarrow P_2 \alpha v_1 - P_2 \alpha c - P_2 k + v_1 - P_2 v_1 - c + P_2 c - k + P_2 k \geq 0$$

$$\Rightarrow v_1(P_2\alpha + 1 - P_2) + c(-P_2\alpha - 1 + P_2) - k \geq 0$$

$$\Rightarrow v_1 \geq c + \frac{k}{P_2\alpha + 1 - P_2}$$

$$\Rightarrow v_1^* = c + \frac{k}{P_2\alpha + 1 - P_2}$$

Substituting $P_2 = 1 - F(v_2^*) = 1 - v_2^*$ when $s = 1$:

$$v_1^* = c + \frac{k}{(1 - v_2^*)\alpha + 1 - (1 - v_2^*)}$$

$$\Rightarrow v_1^* = c + \frac{k}{\alpha + v_2^*(1-\alpha)} \tag{1}$$

Repeating the same method for VC2, we get:

$$P_1[(1-\alpha)(v_2 - c) - k] + (1 - P_1)[v_2 - c - k] \geq 0$$

$$\Rightarrow v_2(1 - \alpha P_1) + c(\alpha P_1 - 1) - k \geq 0$$

$$\Rightarrow v_2 \geq \frac{k + c(1 - \alpha P_1)}{1 - \alpha P_1}$$

Substituting $P_1 = 1 - F(v_1^*) = 1 - v_1^*$ when $s = 1$,

$$v_2^* = c + \frac{k}{1 - \alpha(1 - v_1^*)} \tag{2}$$

Note: Due to the computational difficulty, an online equation solver was used to derive the solutions of v_1^* and v_2^* .

$$v_1^* = \frac{-a^2c^2 + a^2 + ac^2 + 2ac + 2ak - a - c - k + \sqrt{\begin{matrix} a^4c^4 - 4a^4c^3 + 6a^4c^2 - 4a^4c + a^4 - 2a^3c^4 + 8a^3c^3 - 12a^3c^2 + 8a^3c - 2a^3 + a^2c^4 \\ -6a^2c^3 \\ -2a^2c^2k + 10v^2c^2 + 4a^2ck - 6a^2c + 4a^2k^2 - 2a^2k + a^2 + 2ac^3 + 2ac^2k - 4ac^2 - 4ack \\ +2ac - 4ak^2 + 2ak + c^2 + 2ck + k^2 \end{matrix}}}{2a(-ac + a + c)}$$

$$v_2^* = \frac{-a^2c^2 + a^2 + ac^2 - 2ac - 2ak - a + c + k + \sqrt{\begin{matrix} a^4c^4 - 4a^4c^3 + 6a^4c^2 - 4a^4c + a^4 - 2a^3c^4 + 8a^3c^3 - 12a^3c^2 + 8a^3c - 2a^3 + a^2c^4 - 6a^2c^3 \\ -2a^2c^2k + 10a^2c^2 + 4a^2ck - 6a^2c + 4a^2k^2 - 2a^2k + a^2 + 2ac^3 + 2ac^2k - 4ac^2 - 4ack \\ +2ac - 4ak^2 + 2ak + c^2 + 2ck + k^2 \end{matrix}}}{2(-a^2c + a^2 + ac - 2a + 1)}$$

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Given the complexity of the algebraic solutions, analysing the relationship between α and the optimal bid strategy from the solutions above is not very feasible. Nevertheless, the expected results from this game and its impact on investment strategies is as follows.

We know that when α is half, v^* is equal for both venture capitalists and is higher than $c + k$ as per the base model (refer section 4.1). Therefore, we plot a graph of α against v_1^* (the value of VC1, the established venture capitalist) and v_2^* (the value of VC2, the new venture capitalist) between $0 \leq \alpha \leq 1$. It is expected that as α increases v_1^* decreases since the cut-off value they must have in order to win, falls. When α reaches 1, v_1^* has absolute priority, in which case their value is expected to decrease till $(c + k)$ as they win at any value above their cost. On the other hand, v_2^* will perhaps increase as α increases. In this case, increase in α increases the competition for the new venture capitalist. Thus, they would need to have a higher value to enter. Therefore, due to priority, both v_1^* and v_2^* diverge in opposite directions. Its impact on revenue, therefore, would depend on the extent of increase or decrease in the values of VC1 and VC2. If increase in v_1^* is greater than decrease in v_2^*, revenue increases, while the opposite scenario results in a fall in revenue.

3.2 Auction Investment Model

Consider a slightly different model, where 20% of the company is sold, but the amount invested is determined by the bids proposed by the interested venture capitalists. The dilemma arises because they are now both competing for the same asset. This scenario resembles a first-price sealed-bid auction, where the bids are presented in separate sealed envelopes. The seller awards the funding opportunity to the venture capitalist that bids the highest.

Let's say VC1 and VC2 bids values b1, b2 ∈ [0,∞]. Similar to the previous model, VC1's payoff assuming $b_1 > b_2$ is $(v_1 - b_1 - k)$ and $-k$ if they lose the bid. Their own valuations, v_1 and v_2 are uniformly distributed along a function R $[0,\ s^*]$ where $R = \frac{x}{s^*}$.

The next step is to look for an equilibrium where each VC uses a bid strategy that is a strictly increasing, continuous and differentiable function of their value. To achieve this, assume that bidders use identical bidding strategies $b_1 = b_2$. Suppose the venture capitalist, VC bids v according to their true type, but in reality, there is a deviation in their bidding strategy represented by z due to their subjective beliefs. Therefore, assuming s*$= 1$ for the sake of notational simplicity, the probability of the VC bidding is z.

$$\pi(z, v) = z(v - b(z))$$
$$\frac{d\pi}{dz} = z(-b'(z)) + (v - b(z))$$

However, herein lies a contradiction. We are presuming that this equation holds true only in an equilibrium, where VC of type v would not want to deviate from its strategy, $b(v)$. However, since there exists a deviation to type z, it must be true that $z = v$. Therefore, at $z = v$, the VC's expected utility is maximized.

$$v(-b'(v)) + (v - b(v)) = 0$$
$$b'(v) = \frac{v - b(v)}{v}$$

Solving this differential equation, we arrive at

$$b(v) = \frac{v}{2} + \frac{\delta}{v}$$

The expected utility of the lowest type entering the auction is $v(v - 0) - k = 0$ where v is the probability of entering, and their bid and expected value equals zero. Thus, the lowest bid is $\sqrt{k}$.

Substituting $v = \sqrt{k}$,

$$b(v) = \frac{v^2 - k}{2v}$$
$$b(v) = \frac{v}{2} - \frac{k}{2v}$$

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ISSN 2688-3651 [online]

Any firm that enters the market must have a bid $\geq \sqrt{k}$ given that the expected utility for firms that do not enter equals k. Thus, the lowest type entering the auction would have a bid closest to 0, and as the bid value b(v) increases, the expected utility increases at a higher rate. Geometrically speaking, this is represented by an increasing, upward sloping curve.

Total Revenue Generated in the Auction Model

The expected payment of each bidder, $g(v) = b(v) \times v$, is conditional on their type. However, since the lowest type bids $\sqrt{k}$ and $s = 1$, the expected revenue before they know their type can be represented by the definite integral:

$$\int_{\sqrt{k}}^{1} g(v)|dv = \int_{\sqrt{k}}^{1} b(v) \times v$$

After substituting $b(v) = \frac{v^2-k}{2v}$,

$$\int_{\sqrt{k}}^{1} g(v)|dv = \int_{\sqrt{k}}^{1} \frac{v^2-k}{2} \text{ dv} = \frac{1}{2}\int_{\sqrt{k}}^{1} \left(\frac{v^2}{2}\right) dv - \frac{k}{2}\int_{\sqrt{k}}^{1} (1) dv$$

$$\int_{\sqrt{k}}^{1} g(v)|dv = \frac{1}{2}\left(\frac{1}{3} - \frac{k^{\frac{3}{2}}}{3}\right) - \frac{k}{2}\left(1-\sqrt{k}\right)$$

$$\int_{\sqrt{k}}^{1} g(v)|dv = \frac{2k^{\frac{3}{2}} - 3k + 1}{6}$$

Given that this is a 2-player game, the expected revenue is:

$$2 \times \int_{\sqrt{k}}^{1} g(v)|dv = R_a = \frac{2k^{\frac{3}{2}} - 3k + 1}{3}$$

If there are n players, the expected revenue is given by:

$$n \times \int_{\sqrt{k}}^{1} g(v)|dv = \frac{n(2k^{\frac{3}{2}} - 3k + 1)}{6}$$

Therefore, we realise that the expected revenue increases as the number of players increases. Although the start-up only accepts one investor, as the number of players increases, so does the competition, and therefore the venture capitalists would bid higher values to increase their probability of winning.

Calculating Miscoordination in the Auction Model

The bid preparation costs introduce the possibility of miscoordination. Since $k < 1$, $\sqrt{k} > k$, any VC with a value between k and $\sqrt{k}$ would choose not to invest due to the fear of wasting their bid preparation efforts, k. Using the same method as section 4.1.2, we can model this as:

$$M_a = 2\text{k}(\sqrt{\text{k}} - \text{k}) + (\sqrt{\text{k}} - \text{k})^2$$
$$M_a = 2\text{k}\sqrt{\text{k}} - 2\text{k}^2 + \text{k} - 2\text{k}\sqrt{\text{k}} + \text{k}^2 = \text{k} - \text{k}^2$$

Plotting the graph of E_a against k, as shown below, we realise that miscoordination is minimized when k is closest to zero and one when it is not possible to have a value between $\sqrt{\text{k}}$ and k. At k=0.5, miscoordination is at its maximum since the probability of at least one VC having a value between 0 and 1 is the highest.

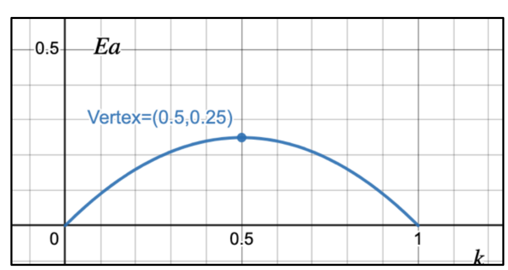

Figure 3: Miscoordination the auction model

Role of Priority in the Auction Model

Priorities in auctions are offered through bidding credits that refer to additional monetary advantages for the established VC given by the start-up in addition to the VC's expected value. The

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ISSN 2688-3651 [online]

established VC1 wins if $b_1 + z > b_2$ where z is the bid credit. In such a situation, there would be two types of competing venture capitalists:

1) those who bid 0 knowing that they will win against any type who does not enter.
2) those who raise their bid to a value, $b_2 \geq b_1 + z$ as there is no benefit from bidding a value less than $b_1 + z$.

Bidding credits majorly serve to increase participation and competition in the auction. When discussing the impact of bid credits on revenues, Ayres and Cramton (1996) show that bid credits increase the competitive pressure, encouraging venture capitalists to bid aggressively. Higher bids would result in greater revenue generation. They support their theory with data from regional auctions, according to which the prices paid for regional licences with bidding credits were 6.2% higher than the prices in standard auctions without bid credits.

While bidding credits are expected to increase revenue, the risk lies in the possibility of overbidding as aggressive bidding may lead to inflated prices.

4. Comparison Between Posted-Price Model and Auction Model

4.1 Comparing Total Revenue

We know that in the auction model, the expected revenue $R_a = \frac{2k^{\frac{3}{2}} - 3k + 1}{3}$.

Within posted price mechanism, let's assume $c = c^* = 0.266$ and $s = 1$.

$$R_p = 0.266\left[1 - \left(\frac{0.266 - 1 + \sqrt{(0.266+1)^2 + 8k}}{2}\right)^2\right]$$

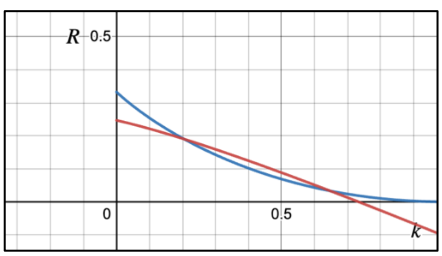

Figure 4: Comparing total revenue generated in auctions and posted price models.

With an increase in k, the bid preparation costs increase, thereby increasing the cost of investment and decreasing the expected profits in both models. However, the rate of decrease in revenue, denoted by the slope of the curves, is higher in the posted-price model than the auction model. R reaches 0 at only $k = 0.734$ in the posted price model, whereas $R = 0$ at $k = 1$ in the auction. A possible explanation for this result lies in the relative differences in elasticities of the two pricing mechanisms. Auctions are characterised by dynamic pricing with variable quantity, in which case venture capitalists have the flexibility to decrease their bid or their quantity demanded. In contrast, posted prices involve fixed, non-negotiable prices set by the seller. In such cases, venture capitalists may become more apprehensive about investing as k increases, making their demand relatively elastic. Therefore, an increase in bid preparation costs is likely to result in a greater fall in revenue within the posted price model than the auction model.

In the posted price model, the revenue generated depends on c. However, in the auction model, the revenue may be greater or smaller depending on the valuation of the start-up by each venture capitalist. If this is less than c, the firm is likely to benefit from a posted price model assuming the VCs invest. If it is greater than c, the start-up may benefit from the auction model.

4.2 Comparing Miscoordination

Bid preparation costs increase the magnitude of miscoordination in the auction model to 0.25 compared to only 0.0875 in the posted price model. Moreover, the inefficiency is minimized to zero at $k = 0.75$ in the posted price system, compared to k = 1 in the auction system.

In both models, we notice that the miscoordination is maximized at intermediate values of k. This suggests that there is a trade-off between the cost of bid preparation and the risk of miscoordination.

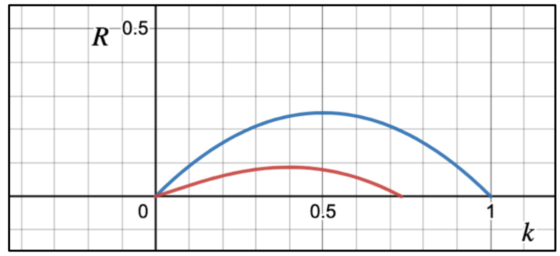

Figure 5: Comparison of miscoordination in both models

Moreover, by calculating the total area of miscoordination from Fig 5, we notice that contrary to initial expectations, miscoordination is greater in the auction than in the fixed price system. Thus, social surplus or welfare loss seems to be higher in auctions as compared to posted price set ups. Irrespective of whether the c value is 1 or 0, the area under the red curve remains lower than that of the blue curve. This suggests that inefficiency is higher in auctions for most values of c. Therefore, the chances of 'falling through the cracks' is lesser in posted price mechanisms, making this a preferred choice for start-ups and entrepreneurs in the venture capital industry.

5. Discussion and Extension

Unfortunately, many factors were not considered to simplify the models. It is assumed that the bidders use identical strategies in the auction model, which is rarely applicable to a true venture capital market. Secondly, in section 5.1, the assumption was that $c = c^* = 0.266$. However, there is a possibility that c^* may differ with changes in the k value, in which case the same results would not apply. Thirdly, the effect of priority through bid credits in the auction model could not be mathematically explored due to its inherent complexity. Fourthly, uniform distribution was assumed throughout for the sake of simplicity, which again limits the potential applications of these results.

Therefore, a reasonable extension would be to include the effect of bid credits with preparation costs in an auction model. Furthermore, there is often a relationship between the bid of one venture capitalist and their assumption of the 'type' of the other player and their risk behaviour. Although this paper assumed risk neutrality, an extension could explore the role of risk 'types' (high risk and low risk) on the bidding strategy of the other venture capitalists.

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