Journal of Research High School

Volume 2021 (2) 1 - 6
ISSN 2688-3651 [online]

Selected Mathematical Models in Hot Air Drying of Foods

Ömer Burak Çınar[1*]

[1] Private Uğur High School, Kahramanmaraş, Turkey

Received March 6, 2021; Revised July 19, 2021; Accepted, July 23, 2021

Abstract

Food processing and preservation are major areas of interest for the food industry. Agricultural products having high amounts of water are highly perishable. Therefore, preservation methods are used to extend their shelf-lives. Drying is one of the oldest food preservation techniques. Among wide range of drying methods, hot air drying is one of the most frequently used drying methods for foods contain high amounts of water. Mathematical models for thin layer drying are useful in explaining heat and mass transfer mechanism and drying rate as well as predicting effective diffusion coefficient and activation energy of the drying process. The aim of the present study is to give fundamental information on both heat and mass transfer mechanisms in hot air drying of foods and selected mathematical models by deriving equations for predicting drying kinetics. It is expected that the present study provides basic background knowledge on hot air drying studies for the researchers.

Keywords: Drying, Heat, Mass, Diffusivity, Moisture Content, Model

1. Introduction

Due to rapid growing world population, global warming and climate change, future projection of the limited food sources has been a great interest for the countries around the world. Many underdeveloped countries are in need of clean water and food supply whereas many others nearly face aridness and desertification. These facts bring researchers and governments to pay close attention and take proper actions especially on agricultural productions for future. High quality future foods are demanded and expected to be more nutritious, safe, natural, organic, functional, shelf stable as well as tasty. All these expectations could be matched to a certain degree with the benefits of food processing. Agricultural products are essential in human nutrition providing bioactive components, water, carbohydrates, proteins, fat/oils, vitamins and minerals necessary for metabolic activities. Because of their highly perishable nature, they need to be preserved by using either thermal or non-thermal processing techniques such as drying, canning, freezing, cooling, smoking, salting, acidifying, microwaving, ohmic heating, dielectric heating, radio frequency processing etc.

Proper preservation technique is expected to lower water activity to a critical level where microbial, chemical and enzymatic control of food is possible meanwhile maintaining mechanical, textural, nutritional and physical properties of food (Arslan and Özcan, 2011). Among those techniques, drying is one of the oldest methods of food preservation (Orikasa et.al., 2008). There has been a growing interest in new and improved drying techniques over

* Corresponding Author
oburakcinar@gmail.com

Advisor: Dr. İnci Doğan
icinar@ksu.edu.tr

the past years. Although recent studies involve non-thermal drying methods as well as hybrid drying techniques, food industry still relies mainly on hot air drying. Hot air drying is a process where water removal from the food material is provided by the pre-heated air flow having constant velocity and relative humidity.

Drying process provides extended shelf life by lowering the water content and water activity of food being dried (Zhou et.al., 2019). Wide range of agricultural products are preserved by drying but dried products display altered organoleptic and nutritional properties as compared to their fresh counterparts. Moisture content of dried food is below its critical value where enzymatic, chemical and biochemical reactions are limited. Convective hot air drying is one of the most frequently used drying methods for food drying operations both in lab and industrial scale. Hot air drying process helps reduce product volume, facilitates handling and transportation, provides microbial, enzymatic and chemical stabilization of dried food, lowers storage expenses and offers a variety of processed foods to consumer (Zielinska and Markowski, 2010). Hot air drying also has some drawbacks such as longer drying time and undesired changes of food due to its heat sensitivity (Darvishi et.al., 2014). Studies on hot air drying of foodstuffs concentrate on the optimization of drying parameters, namely drying temperature, time, air velocity, air and food relative humidity, shrinkage, thickness, evaporation surface area and moisture content of food and type of drying equipment. All of these factors in return influence the rate of drying (Cruz et.al., 2015; Guine, 2018).

The aim of the present study is to give fundamental information on both heat and mass transfer mechanisms in hot air drying of foods and selected mathematical models by deriving equations for predicting drying kinetics. It is expected that the present study provides basic background knowledge on hot air drying studies for researchers.

2. Mechanisms of Heat and Mass Transfer

The purpose of drying is the water removal from the food. Drying involves simultaneous heat and mass transfer for water removal. During hot air drying, two simultaneous process occurs namely heat transfer from the heated drying air to the food surface and moisture (mass) transfer from the surface of the solid food to the hot drying air. Driving forces for mass transfer are capillary diffusion, concentration difference and pressure difference between food and hot drying air whereas driving forces of heat transfer are temperature difference and thermal conductivity of the drying food.

Drying behavior of food material is affected by internal (density, permeability, porosity, sorption characteristics, thermophysical properties of food) and external (temperature, velocity and relative humidity of heated drying air) factors (Kaya et.al., 2009). Knowledge on heat and mass transfer phenomena during hot air drying of foods greatly helps to reduce operational costs and preserve the quality of dried food via drying parameter optimisation studies.

Theoretical and experimental studies on heat and mass transfer phenomena during hot air drying of foods has been a major research area over the past decades. Diffusion theory, the capillary flow theory and evaporation-condensation theory are widely used to explain the physical phenomena of the hot air drying process in porous materials (Prommas, 2011).

3. Mathematical Models for hot air drying

From the food engineering point of view, mathematical models are used as effective tools for describing drying mechanisms for process optimisation and analysis of food drying for wide range of foods. Fundamental understanding of drying characteristics is generally build on thin layer drying concept and models are theoretical, semi-theoretical or purely empirical nature (Mwithiga & Olwal, 2005). Thin layer drying is based on placing drying food onto drier trays as thin layered as possible to ease water removal from the surface of the food during drying. Most of the studies from literature use semi-theoretical models with a certain degree of success depending on the type of food and conditions of drying. Advantage of using of semi-theoretical models is in their less complicated solutions as compared to the theoretical models (Doungporn et.al., 2012). Most commonly used semi-theoretical thin layer drying models from the selected literature are given in Table 1.

Table 1. Semi-theoretical thin layer drying model equations

Name of model	Model equation MR=	Reference
Lewis Newton	exp(-kt)	Artnaseaw et.al., 2010; Mwithiga&Olwal, 2005
Page	exp(-kt^n)	Artnaseaw et.al., 2010
Modified Page	exp(-(kt)^n) exp[-c(t/L^2)^n]	Özdemir&Devres, 1999; Mwithiga &Olwal, 2005 Diamante&Munro, 1991
Henderson and Habis	a exp(-kt)	Henderson&Habis, 1961
Modified Henderson and Habis	a exp(-kt) + b exp(-gt) +c exp(-ht)	Karathanos, 1999
Logarithmic	a exp(-kt) + c	Yaldız et.al., 2001
Two-term	a exp(-k$_o$t) + b exp(-k$_1$t)	Artnaseaw et.al., 2010
Two-term exponential	a exp(-kt) + (1-a) exp(-kat)	Sharaf-Eldeen et.al., 1980
Midilli and Kucuk	a exp(-kt^n) + bt	Midilli et.al, 2002
Approximation of diffusion	a exp(-kt) + (1-a) exp(-kbt)	Demir et.al., 2007
Wang and Singh	1 + at + bt^2	Wang&Singh, 1978
Simplified Fick's diffusion	a exp[-c(t/L^2)]	Diamante&Munro, 1991;
Thomson	t=a ln(MR) + b(ln(MR))^2	Paulsen &Thomson, 1973
Verma et.al.	a exp(-kt) + (1-a) exp(-gt)	Verma et.al., 1985

a,b,c,n and k are model constants.

Most of the studies on mathematical modelling of hot air drying focus on the semi-theoretical thin layer drying. Experimental data fits onto the selected model equations to find the best fit under constant drying conditions. Moisture ratio of samples during drying is calculated as (Guine et.al., 2011; Mota et.al., 2010):

$$MR = \frac{M_t - M_e}{M_0 - M_e} \quad 1$$

where M is the moisture content of the food (kg water/ kg DM), subscripts t, 0 and e denotes time t, initial and equilibrium respectively. M_e refers to the equilibrium of moisture content with surrounding medium namely drying air for a long period of drying time. For most drying operations M_e is relatively small as compared to M_t and M_0 and therefore can be ignored for long period of drying time. Then, Equation 1 becomes:

$$MR = \frac{M_t}{M_0} \quad 2$$

Considering diffusion is, as in most case, the main mechanism for the mass transfer from the drying food surface, solution of Fick's second law of diffusion equation for a slab with no shrinkage can be given as (Singh and Pandey, 2012):

$$MR = \frac{M_t - M_e}{M_0 - M_e} = \sum_{n=1}^{\infty} \left(\frac{8}{(2n-1)\pi^2} exp \left(\frac{-D_{eff}(2n-1)^2 \pi^2}{4(L)^2} t \right) \right) \quad 3$$

where D_{eff}: effective moisture diffusivity (m^2/s); L:half thickness of the slab (m), i: positive integer, number of terms that is taken into account and t: drying time (s). Using first term of series in Equation 3, moisture ratio is simplified to Equation 4:

$$MR = \frac{M_t - M_e}{M_0 - M_e} = \frac{8}{\pi^2} \exp exp \left(\frac{\pi^2 D_{eff}}{4L^2} t \right) \quad 4$$

Equation 4 then can be rearranged for further simplification as in Henderson and Habis model and given as follows:

$$MR = \frac{M_t - M_e}{M_0 - M_e} = a\ exp(-kt) \quad 5$$

Fick's law to explain moisture diffusion process also can be given for a cube geometry of food in series type of equation as (Zielinska and Markowski, 2010):

$$MR = \frac{512}{\pi^6} \left[\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} exp(-(2n-1)kt)^2 \right] \quad 6$$

where n: integer and k: model constant. Moisture ratio can further be simplified using the first term of series in Eqation 6 as follow:

$$MR = \frac{512}{\pi^6}\left[exp(-kt)^2\right] \qquad 7$$

Analytical solution of Fick's second law of diffusion in spherical geometry can be given as (Aghbashlo et.al., 2008):

$$MR = \frac{M_t-M_e}{M_0-M_e} = \frac{6}{\pi^2}\exp\sum_{n=1}^{\infty}\frac{1}{n^2}exp\left(\frac{-n^2\pi^2 D_{eff}}{{r_0}^2}t\right) \qquad 8$$

where r_0 is the radius of the food material (m). When first term of the series Equation 8 is used:

$$MR = \frac{M_t-M_e}{M_0-M_e} = \frac{6}{\pi^2}\exp exp\left(\frac{-\pi^2 D_{eff}}{{r_0}^2}t\right) \qquad 9$$

To simplify Equation 9 to a straight line equation:

$$\ln ln\ (MR) = \ln ln\ \left(\frac{6}{\pi^2}\right) - \left(\frac{\pi^2 D_{eff}}{{r_0}^2}\right)t = A + Bt \qquad 10$$

where A=6/ π^2 ; B= $\pi^2 D_{eff}/{r_0}^2$, slope of the straight line by which D_{eff} can be calculated.

Experimental data of MR and t is used to fit to model equations and goodness of the fit is determined by statistical analysis such as determination coefficient (R), correlation coefficient (R^2), standart error of estimate, sum of squares, mean of squares, F test and P value (Guine et.al., 20111; Mota et.al, 2010). Statistical analysis on selection of best fit hot air drying equation is generally based on correlation coefficient (R^2), reduced Chi-square (χ^2), root mean square error (RMSE) and mean bias error (MBE) as respectively follow (Evin, 2012):

$$\chi^2 = \frac{\sum_{i=1}^{N}(MR_{exp,i}-MR_{pre,i})^2}{N-n} \qquad 11$$

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2\right]^{1/2} \qquad 12$$

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2 \qquad 13$$

where $MR_{exp,i}$: ith moisture content recorded experimentally; $MR_{pre,i}$:ith predicted moisture content by mathematical model in question; N: total number of recorded moisture content values,n: model constant. Best fit model is decided by having the highest R^2 and the lowest χ^2, RMSE and MBE values.

Drying rate is also one of the important parameters in comparisons of drying operations. During hot air drying, moisture removal occurs at different rates namely constant rate and falling rate. For highly moist material as in most foods, both periods are present. Constant rate period is defined as the period of drying where the rate of moisture transfer from the center of the food to the evaporation surface is equal to the rate of moisture evaporation from the surface. Constant rate period continues as long as water supplied to the surface compensates the surface evaporation. Falling rate corresponds to the period where drying rate starts to decrease due to decreasing transfer of water to the evaporation surface. Drying rates of food can be calculated as:

$$DR = \frac{M_{t+\Delta t}-M_t}{\Delta t} \qquad 14$$

where DR: drying rate (kg water/kg DM min); $M_{t+\Delta t}$: the moisture content at time t+Δt and M_t: moisture content at time t (min).

4. Calculation of Effective Moisture Diffusivity and Activation Energy

Water diffusion is a complex phenomena during drying involving molecular diffusion, hydrodynamic, capillary and Knudsen flow and surface tension (Erbay & İcier, 2009). Lumped parameter model assumes only effective moisture diffusivity which technically corresponds to the conductive term of moisture transfer mechanism. Effective diffusivity is generally calculated by the use of experimental drying curve plots. From the experimental data, the plot of ln(MR) versus drying time (t) gives a straight line and slope of this line is:

$$\ln ln\ (MR) = \ln ln\ (a) - \frac{\pi^2 D_{eff}}{4L^2}t \qquad 15$$

where D_{eff}: effective moisture diffusivity (m^2/s); k: drying rate constant, slope of the ln(MR) versus t.

Effective diffusivity is affected both by internal conditions of food namely temperature, moisture content and microstructure and external conditions namely drying air velocity, air temperature and relative humidity. Temperature dependency of D_{eff} is often constructed with Arrhenius type relation as follows (Fudholi et.al, 2016):

$$D_{eff} = D_0 \exp exp \ - \frac{E_a}{RT} \quad 16$$

Equation 16 can be linearized by taking natural logarithms of both sides:

$$ln(D_{eff}) = ln(D_0) - (\frac{E_a}{R})\frac{1}{T} \quad 17$$

where D_0:effective diffusivity at infinitely high temperature (m^2/s), Ea: activation energy (kJ/mol), R: universal gas constant (8.314 $x10^{-3}$kJ/molK); T: absolute temperature (K). E_a and D_0 in Equation 17 can be calculated by the slope of the plot of ln(Deff) versus 1/T.

Conclusion

Rapid increasing world population is in need of nutritious, safe, standardized and yet tasty foods with acceptable shelf-life. Food processing industry is constantly seeking new and improved methods for food processing and preservation. Drying has been and still is one of the most widely used method of food preservation and mainly relies on simultaneous transfer of heat and mass. Fundamentals of heat and mass transfer mechanisms help to improve present drying techniques as well as gives insight for innovations. Mathematical models are used to describe both water removal and heat penetration during hot air drying. Mathematical models for hot air drying of foods are used to develop new drying equipment, drying method and process parameters. Models are tools for process control. Considering rapid developments in artificial intelligence systems, such mathematical models have great potential for further integration with hands-on industrial drying operations for better process and product control. Present work is considered as useful summary for the students set their mind on getting engineering education.

References

Arslan, D., & Özcan, M.M. (2011). Dehydration of red bell-pepper (*Capsicum annuum L.*): Change in drying behavior, colour and antioxidant content. *Food and Bioproduct Processing*, 89:504-511.

Artnaseaw, A., Theerakulpisut, S., & Benjapiyaporn C. (2010). Drying characteristics of Shiitake mushrooms and Jinda chili during vacuum heat pump drying. *Food and Bioproducts Processing*, 88:105-114.

Cruz, A.C.R., Guine, P.F. & Gonçalves, J.C. (2015). Drying kinetics and product quality for convective drying of apples (cvs. Golden Delicious and Granny Smith). *International Journal of Fruit Science*, 15(1):54–78.

Darvishi, H., et al. (2014). Study of the drying kinetics of pepper. *Journal of the Saudi Society of Agricultural Sciences*, 13:130-138.

Demir, V., Gunhan, T. & Yagcıoglu, A.K. (2007). Mathematical modelling of convection drying of gren table olives. *Biosystems Engineering*, 98:47-53.

Diamante, L.M. & Munro, P.A. (1991). Mathematical modelling of hot air drying of sweet potato slices. *International Journal of Food Science and Technology*, 26:99-103.

Doungporn, S., Poomsa-ad, N. & Wiset, L. (2012). Drying equations of Thai Hom Mali paddy by using hot air, carbon dioxide and nitrogen gases as drying media. *Food and Bioproducts Processing*, 90(2):187-198.

Erbay, Z. & İcier, F. (2009). A review of thin layer drying of foods: theory, modeling and experimental results. *Critical Reviews in Food Science and Nutrition*, 50:441-464.

Evin, D. (2012). Thin layer drying kinetics of Gundelia tournefortii L. *Food and Bioproducts Processing*, 90(2):323-332.

Fudholi, A., et al. (2016). Activation energy of thin layer drying kinetics of belimbing dayat fruit (*Baccaurea angulata*). *Research Journal of Applied Sciences, Engineering and Technology*, 13(4):295-300.

Guine, R.P.F. (2018). The drying of foods and its effect on the physicochemical, sensorial and nutritional properties. *International Journal of Food Engineering*, 4(2):93-100.

Guine, R.P.F., Pinho, S. & Barroca, M.J. (2011). Study of the convective drying of pumpkin

(Cucurbita maxima). *Food and Bioproducts Processing*, 89:422-428.

Henderson, S.M., & Habis, S. (1961). Grain drying theory. I. Temperature effect on drying coefficient. *Journal of Agricultural Research and Engineering*, 6:169-174.

Karathanos, V.T. (1999). Determination of water content of dried fruits by drying kinetics. *Journal of Food Engineering*, 39:337-344.

Kaya, A., Aydın, O. & Demirtaş, C. (2009). Experimental and theoretical analysis of drying carrots. *Desalination*, 237:285-295.

Midilli, A., Kucuk, H. & Yapar, Z. (2002). A new model for single-layer drying. *Drying Technology*, 20(7):1503-1513.

Mota, C.L., et al. (2010). Convective drying of onion: kinetics and nutritional evaluation. *Food and Bioproducts Processing*, 88:115-123.

Mwithiga, G., & Olwal, J.O. (2005). The drying kinetics of kale (Brassica oleracea) in a convective hot air drier, *Journal of Food Engineering*, 71:373-378.

Orikasa, T., et al. (2008). Drying characteristics of kiwi fruit during hot air drying. *Journal of Food Engineering*, 85(2):303-308.

Özdemir, M., & Devres, Y.O. (1999). Thin layer drying characteristics of hazelnuts during roasting. Journal of Food Engineering, 42:225-233.

Paulsen, M.R. &Thomson, T.L. (1973). Drying endysus of grain sorghum. *Transactions of the ASAE*, 16:537-540.

Prommas, R. (2011). Theoretical and experimental study of heat and mass transfer mechanism during convective drying of multi-layered porous packed bed. *International Communications in Heat and Mass Transfer*, 38:900-905.

Sharaf-Eldeen, Y.I.,Blaisdell, J.L. & Hamdy, M.Y. (1980). A model for ear corn drying. *Transactions of the ASAE*, 5:1261-1265.

Singh, N.J. & Pandey, R.K. (2012). Convective air drying characteristics of sweet potato cube (Ipomoea batatas L.). Food and Bioproducts Processing, 90(2):317-322.

Verma, L.R., et al. (1985). Effects of drying air parameters on rice drying models. *Transactions of the ASAE*, 28:296-301.

Wang, C.Y. & Singh, R.P. (1978). Use of variable equilibrium moisture content in modelling rice drying. *Transactions of American Society of Agricultural Engineers*, 11:668-672.

Yaldız, O., Ertekin, C., & Uzun, H.I. (2001). Mathematical modelling of thin layer solar drying of sultana grapes. *Energy*, 26:457-465.

Zhou, X., et al. (2019). Combined radio frequency-vacuum and hot air drying of kiwifruits: Effect on drying uniformity, energy efficiency and product quality. *Innovative Food Science and Emerging Technologies*, https://doi.org/10.1016/j.ifset.2019.102182.

Zielinska, M. & Markowski, M. (2010). Air drying characteristics and moisture diffusivity of carrots. *Chemical Engineering and Processing: Process Intensification*, 49:212-218.