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Centipede's Leap into the Quantum Realm

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Abstract

The centipede game is a two-player non-zero-sum game. Each turn, a player can choose whether they want to take or pass a growing reward. The classical, rational solution of this game shows defection in the first round, when in reality, players cooperate much more often. Inspired by prior work employing quantum strategies in the prisoner's dilemma, this paper applies the Eisert-Wilkens-Lewenstein (EWL) protocol to a 3-round centipede game and finds two degenerate Nash equilibria that give both players the optimal payoff of (2,2), which is better than if the players followed classical backward induction. Intriguingly, the probability of defection in the last round is exactly zero for both cases, breaking the backward-induction dilemma. Simulations on Qiskit support the theoretical outcomes. These findings suggest that quantum algorithms can reach cooperative outcomes and more accurately model observed human behavior.

Keywords: Quantum game theory, Backward induction, Entanglement, Centipede game, Nash equilibrium, Non-zero-sum game

1. Introduction

Game theory has a wide range of applications in various fields such as economics, law, evolutionary biology, and computer science. This paper focuses on the algorithmic side of game theory, particularly in its application to analyzing decisions and strategies. The two main categories of games in which game theory is applied are cooperative and non-cooperative games. This study's emphasis will be on non-cooperative games, where each player chooses their strategy independently of others. An essential component in these games is that each player assumes that the other player(s) will act rationally.

The first official research paper, *Zur Theorie der Gesellschaftsspiele*, about classical game theory, was published by John von Neumann (1928). A key idea that von Neumann introduced was the Minimax theorem. The Minimax theorem proves that in every two-player zero-sum game, an equilibrium can be reached if both players use mixed strategies. The minimax theorem inspired George Dantzig to coin the term linear programming in 1947 (Roughgarden, 2016). Dantzig then created the simplex method, which was a prevalent algorithm to solve linear programming problems, especially concerning optimization (Sekhon & Bloom, 2020). The simplex method was used for a variety of optimization problems, most notably in the US Air Force during WW2 (Forder, 1998). In 1964, Carlton Lemke and Joseph Howson built on the pivoting structure of the simplex method. However, instead of using it for optimization problems, Lemke and Howson used it for finding the Nash equilibrium in a non-zero-sum two-player game (Lemke & Howson, 1964). Nash equilibrium is the idea that changing one's strategy won't result in a better personal outcome (Holt & Roth, 2004). It is considered to be the most stable outcome of a strategic interaction. These developments laid the foundation for classical game theory.

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Recent decades have seen the emergence of quantum mechanics as a powerful tool for re-imagining these models. Quantum mechanics emerged in the early 20th century to explain phenomena that classical physics couldn't, such as blackbody radiation and the photoelectric effect (Styer, 1999). Many key findings made by scientists like Schrodinger, Einstein, and Heisenberg led to a new mathematical framework for describing particles at the sub-atomic level, which was necessary for advancements in quantum computing. The original idea that quantum computers could be superior to their classical counterparts came from Richard P. Feynman (Feynman, 1982), who argued that quantum systems couldn't be effectively represented by classical computers. He introduced the concept of quantum computers as a necessary tool for modeling nature. Scientists were eventually able to build on his work and produce core ideas, such as superposition and entanglement. These ideas are now being applied to rethink decision-making processes in fields like economics, cryptography, and game theory.

Quantum game theory is an extension of classical game theory, incorporating concepts from quantum mechanics (entanglement, unitary operations, tensor products, circuits), to rethink strategic decisions in both zero-sum and non-zero-sum games (Ghosh, 2021). Instead of using discrete or probabilistic choices, players use qubits to evaluate multiple strategies at once with superposition (modeled with qubits being in two states at once) and apply unitary operations (representations of players' decisions) as strategies. Entanglement can be used to model how one player's choice can instantly affect the strategy of others, even without communication, and tensor products can be used to model the full state of a game, with all players' strategies represented.

In the past, many studies have examined quantum strategies across all types of games. One of the first papers on quantum strategy was written by David A Meyer. Meyer mainly explored the game theory of the quantum penny flip game, a zero-sum game (Meyer, 1998). He proved that: "in general a quantum strategy is always at least as good as a classical one". Following Meyer's work, the field of quantum game theory rapidly expanded, showing that the possible strategies players can implement drastically increase when applying the principles of quantum mechanics. Das (2023) and Landsburg (2011) show that new Nash equilibria can be achieved when using quantum strategies and outperform classical models in certain situations.

This shift is relevant in games like the centipede game, a non-zero-sum game where two players take turns deciding whether to pass or take an increasingly large pot of money. The game ends once a player decides to defect and take the money, or when both players cooperate until the end of the game (the number of rounds may vary). Backward induction is an example of a possible strategy to find the Nash equilibrium for the centipede game. Although there are many solutions to find the proposed Nash equilibria for the centipede game, backward induction is often the most famous and referenced one. Backward induction reasons from the end of the game to the start of the game to determine their best course of action (Yildiz, 2012). Backward induction relies on the fact that players will assume that their opponents will always act in their best interest, therefore making the future of the game predictable. Analyzing a classical centipede game using backward induction, a player should always choose to defect and take the money in the first round.

Figure 1 depicts the structure of the centipede game modeled in this paper. Player l(Pl) moves first and can either choose to cooperate or defect, described as the branches labeled c and d, respectively. If Pl defects, the game ends and Pl receives a payoff of 1, while Player 2 (P2) receives a payoff of 0 (payoff pair read as (Payoff for Pl, Payoff for P2)). If Pl cooperates, the game proceeds to round 2 where P2 can choose to defect or cooperate. In the third round, Pl may cooperate to have the final payoff of (2,2), or defect and have the final payoff being (3,1).

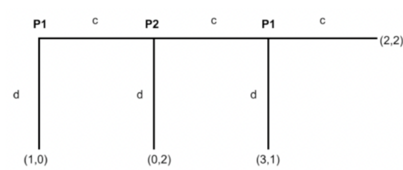

Figure 1: Illustration of the 3 rounds played in the centipede game

Backward induction for the centipede game contains the following procedure: First start with the last round. Since Pl can obtain a payoff of 3 by defecting, Pl would choose to defect in round 3. Then, P2 knows that rationally, Pl would defect at round 3, which would result in P2's payoff being l; thus, P2 would defect at round 2 so that its payoff is 2. Consequently, Pl would predict P2 would defect at round 2, giving Pl a payoff of 0; therefore, Pl will always defect in the first round to get a payoff of 1. The centipede game is known for the gaps between its theoretical predictions and its experimental results, as players in real life do not follow the strategy of backward induction (Elmshauser, 2022; McKelvey & Palfrey, 1992).

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Quantum strategies have been proven to outperform classical alternatives in certain circumstances (Simon, 1994; Ibarrondo et al., 2022; Rao et al., 2024) since they lead to better joint outcomes due to their initial entangled state. The mechanism of entanglement (two qubits are "entangled" such that measuring one qubit instantly affects the state of another) allows for different actions and events to affect each other in a way that is classically unreproducible, leading to more cooperation in different games. An example of this is in the quantum version of the Prisoner's Dilemma (Eisert et al., 1999). The Prisoner's Dilemma is a game in which two players are presented with the option to rat out their partner (defect) or remain silent (cooperate) during an interrogation. If both players defect, they will have minimal punishment. However, if one player chooses to defect and one cooperates, the one who defects doesn't receive a punishment, while the player who cooperates gets a considerable punishment. The study uses the Eisert-Wilkens-Lewenstein (EWL) Protocol, a well-known framework for quantizing games with 2 players and one round. The best result found by Eisert et al. was when both players cooperated.

The EWL Protocol inspires the authors to explore quantization of a more complex game. In particular, while classical backward induction in the centipede game only benefits one player, the authors hypothesize that applying quantum strategies to the centipede game will result in a more cooperative and rewarding outcome for both players, aligning closer with how the game might unfold in real life.

There has been previous work applying quantum mechanics to the centipede game. Frąckiewicz (2017) adopted the Marinatto-Weber scheme (a scheme in which a player can choose to play a classical game or its quantum counterpart) and found a theoretical, pure-strategy equilibrium of (4.5,4.5). However, the equilibrium is not "play-on to the very end"; Each individual run collapses to either (6, 4) or (3, 5); averaging the two classical payoffs with equal probability gives the average of (4.5, 4.5). This study aims to address these drawbacks.

This paper explores how a quantum circuit-based algorithm approaches the Centipede Game differently from classical methods, particularly in terms of outcomes. By modifying the EWL protocol from the prisoner's dilemma (Eisert et at., 1999), the authors find the mathematical solutions to the centipede game. This paper will then model the Centipede Game using Qiskit, a Python-based quantum computing framework, to show the feasibility of such a quantum approach in solving the Centipede Game.

2. Methodology

2.1 Math

This paper adapts the Eisert-Wilkens-Lewenstein (EWL) protocol (Eisert et al., 1999) to the centipede game. The EWL protocol was originally developed to fit a 2-player, 1-round prisoner's dilemma game. The authors made three important changes to the protocol: three qubits are used to model each round, instead of two in the EWL protocol with one for each player; the entanglement of the two qubits, which involves a Bell state, has been replaced by a three-qubit GHZ-type entangled state; the inverse entangler operator $J^\dagger$ is applied before measurement to disentangle the qubits.

To entangle the decisions together, each qubit has to be in a state of superposition. Entanglement connects the three round-qubits together in such a way that no one round is considered separately; any decision made by the players in one round will affect the likelihood of results in other rounds. This is how the quantum strategies stop backward induction, which considers only one round at a time. Mathematically, the J entangler turns unentangled qubits I000) into

$$|\psi_{\text{unentangled}}\rangle = |000\rangle\,, \qquad |\psi_{\text{entangled}}\rangle = \frac{|000\rangle + i\,|111\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,|000\rangle + \frac{1}{\sqrt{2}}\,i\,|111\rangle\,.$$

In circuits, the J entangler can be replicated by applying a Hadamard gate on the first qubit, then two CNOT gates on lst→2nd qubit and 2nd→3rd qubit, and finally an S relative phase gate.

Û is a unitary operator where the player's decision could be encoded as a superposition of both cooperate and defect.

In practice, any single-parameter rotation gate can be set to perform the operation, such as Rx(Θ), Ry(Θ). The

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results would be equivalent regardless of choice. The $Ry(-\Theta) = \hat{U}$ gate was chosen because the J entangler already introduces a relative phase factor, making the ϕ parameter irrelevant. Thus, if $\phi = 0$, the operator will be reduced to real values, simplifying the calculations without loss of generality. $Ry(-\Theta)$ was also the unitary of choice by Eisert et al. (1999).

$$\hat{U}(\theta,\phi) = \begin{bmatrix} e^{i\phi}\cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & e^{-i\phi}\cos(\theta/2) \end{bmatrix}, \qquad \hat{U}\left(\theta,\frac{\pi}{2}\right) = \begin{bmatrix} i\cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & -i\cos(\theta/2) \end{bmatrix}.$$

This is the quantum gate mentioned by EWL. However, since this game is entangled with a relative phase (the i in front of $|111\rangle$), the imaginary number in the matrix is redundant and is equivalent to

$$\hat{U}(\theta,0) = \begin{bmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{bmatrix},$$

which is much easier to use, since it's all in the reals, and this would still represent maximum entanglement.

After applying $\hat{U}$ to the states of cooperation ($|0\rangle$) and defection ($|1\rangle$), the tensor product of all three rounds is taken. The tensor product operation combines the states from the three rounds into a single state that describes the entire game, yielding the probability amplitudes for all possible combinations of game outcomes. The tensor product is necessary because currently, each round's quantum state exists independently, but the game requires the combined probabilities of all three states. It's similar to how a three course menu with 2 options each has 8 possible meals, each with its own probability. To take the tensor product, the probability amplitudes of cooperation and defection are first found to be the following:

$$\alpha = \cos(\theta/2), \qquad \beta = \sin(\theta/2),$$

$$U_1|0\rangle = \alpha_1|0\rangle - \beta_1|1\rangle, \qquad U_2|0\rangle = \alpha_2|0\rangle - \beta_2|1\rangle, \qquad U_3|0\rangle = \alpha_3|0\rangle - \beta_3|1\rangle,$$
$$U_1|1\rangle = \beta_1|0\rangle - \alpha_1|1\rangle, \qquad U_2|1\rangle = \beta_2|0\rangle - \alpha_2|1\rangle, \qquad U_3|1\rangle = \beta_3|0\rangle - \alpha_3|1\rangle.$$

The subscripts show the round number. Each round starts in the default state of cooperation. The alphas and betas represent the probability of each round ending in either cooperation or defection. The subscript on alpha and betas represents the respective θ variable.

The tensor product of the unitary operators to the entangled qubits is:

$$|\psi_{\text{entangled}}\rangle = \frac{1}{\sqrt{2}}\big(U_1|0\rangle \otimes U_2|0\rangle \otimes U_3|0\rangle\big) + \frac{i}{\sqrt{2}}\big(U_1|1\rangle \otimes U_2|1\rangle \otimes U_3|1\rangle\big).$$

1) Expansion of $U_1|0\rangle \otimes U_2|0\rangle \otimes U_3|0\rangle$.

$$\begin{aligned} U_1|0\rangle \otimes U_2|0\rangle \otimes U_3|0\rangle &= (\alpha_1|0\rangle - \beta_1|1\rangle) \otimes (\alpha_2|0\rangle - \beta_2|1\rangle) \otimes (\alpha_3|0\rangle - \beta_3|1\rangle) \\ &= \alpha_1\alpha_2\alpha_3|000\rangle - \alpha_1\alpha_2\beta_3|001\rangle - \alpha_1\beta_2\alpha_3|010\rangle + \alpha_1\beta_2\beta_3|011\rangle \\ &\quad - \beta_1\alpha_2\alpha_3|100\rangle + \beta_1\alpha_2\beta_3|101\rangle + \beta_1\beta_2\alpha_3|110\rangle - \beta_1\beta_2\beta_3|111\rangle. \end{aligned}$$

2) Expansion of $U_1|1\rangle \otimes U_2|1\rangle \otimes U_3|1\rangle$.

$$\begin{aligned} U_1|1\rangle \otimes U_2|1\rangle \otimes U_3|1\rangle &= (\beta_1|0\rangle - \alpha_1|1\rangle) \otimes (\beta_2|0\rangle - \alpha_2|1\rangle) \otimes (\beta_3|0\rangle - \alpha_3|1\rangle) \\ &= \beta_1\beta_2\beta_3|000\rangle - \beta_1\beta_2\alpha_3|001\rangle - \beta_1\alpha_2\beta_3|010\rangle + \beta_1\alpha_2\alpha_3|011\rangle \\ &\quad - \alpha_1\beta_2\beta_3|100\rangle + \alpha_1\beta_2\alpha_3|101\rangle + \alpha_1\alpha_2\beta_3|110\rangle - \alpha_1\alpha_2\alpha_3|111\rangle. \end{aligned}$$

After calculating the tensor products for the probabilities of both $|0\rangle$ and $|1\rangle$, the rounds were disentangled to calculate the probabilities of each outcome. Disentanglement is needed before measurement or else the three qubits will be entangled and unable to measure their individual results. The process of disentanglement is analogous to unbraiding strings to measure each one separately. To disentangle the rounds, $J^\dagger$ was used, and the amplitudes (a) are:

a = *final amplitude after disentanglement*, A = *amplitude during the disentanglement*.

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$$a_{000}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{000} + A_{111})\,, \qquad a_{111}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{111} - A_{000})\,,$$
$$a_{001}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{001} + A_{110})\,, \qquad a_{110}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{110} - A_{001})\,,$$
$$a_{010}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{010} + A_{101})\,, \qquad a_{101}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{101} - A_{010})\,,$$
$$a_{011}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{011} + A_{100})\,, \qquad a_{100}^{\text{out}} = \frac{1}{\sqrt{2}}(A_{100} - A_{011})\,.$$

Eg Amplitude of A_{000}: $A_{000} = (\alpha_1\alpha_2\alpha_3 + \beta_1\beta_2\beta_3)\,|000\rangle$.

Substituting yields:

$$\begin{aligned}|\psi_{\text{unentangled}}\rangle = {} & (\alpha_1\alpha_2\alpha_3 + i\beta_1\beta_2\beta_3)\,|000\rangle \\ & + (\alpha_1\beta_2\beta_3 + i\beta_1\alpha_2\alpha_3)\,|011\rangle \\ & + (\beta_1\alpha_2\beta_3 + i\alpha_1\beta_2\alpha_3)\,|101\rangle \\ & + (\beta_1\beta_2\alpha_3 + i\alpha_1\alpha_2\beta_3)\,|110\rangle\,.\end{aligned}$$

4 key notations are canceled out.

An interpretation of this wave notation is: for any ket notation in the form $|abc\rangle$, a is the probability of defecting at round 1, b is the probability of defecting at round 2, and c is the probability of defecting at round 3. For example $|011\rangle$ means cooperation in round 1 and defection in rounds 2 and 3. These interpretations can be used in the following way:

There are 4 different total probabilities in the 3-round centipede game, P_{1d1}, P_{2d2}, P_{3d1}, P_{3c1}; each of these 4 probabilities can be found using the wave function formula above. For example, P_{1d1} means defection in round 1. So, the authors may add up the probabilities (recall that $P(a) = |a|^2$, a = amplitude) of all of the qubit equivalents: 100, 110, 101, 111. From the final formula, the coefficients of 100 and 111 are 0; thus, only 110 and 101 probabilities need to be added, yielding:

$$(\beta_1\alpha_2\beta_3)^2 + (\alpha_1\beta_2\alpha_3)^2 + (\beta_1\beta_2\alpha_3)^2 + (\alpha_1\alpha_2\beta_3)^2.$$

Using the same logic:

$$\begin{aligned}P_{1d_1} &= (\beta_1\alpha_2\beta_3)^2 + (\alpha_1\beta_2\alpha_3)^2 + (\beta_1\beta_2\alpha_3)^2 + (\alpha_1\alpha_2\beta_3)^2,\\ P_{2d_2} &= (\alpha_1\beta_2\beta_3)^2 + (\beta_1\alpha_2\alpha_3)^2,\\ P_{3d_1} &= 0,\\ P_{3c_1} &= (\alpha_1\alpha_2\alpha_3)^2 + (\beta_1\beta_2\beta_3)^2.\end{aligned}$$

To find the maximum payoff for each player, an equation was made by multiplying the reward of the move by the probability of getting that reward, which was represented by the decision (cooperate or defect), the player, and the round number. The maximum payoff equation of both player 1 and player 2 is shown below:

Player 1: $\$_1 = 1P_{1d_1} + 0P_{2d_2} + 3P_{3d_1} + 2P_{3c_1}.$

Player 2: $\$_2 = 0P_{1d_1} + 2P_{2d_2} + 1P_{3d_1} + 2P_{3c_1}.$

The dollar sign represents the maximum payoff for each player. The coefficient of each variable shows the payoff received by the respective player. The subscript after *P* shows the round number, and the c's in rounds one and two weren't included during the calculations because there isn't a payoff for cooperating. Recall that the probability of defecting in the 3rd round was 0. Therefore, plugging 0 in for P_{3d1}, the equation for the total payoff can be simplified like below:

Player 1: $\$_1 = 1P_{1d_1} + (0)P_{2d_2} + 3(0) + 2P_{3c_1} = P_{1d_1} + 2P_{3c_1}.$

Player 2: $\$_2 = 0P_{1d_1} + 2P_{2d_2} + 1(0) + 2P_{3c_1} = 2P_{2d_2} + 2P_{3c_1}.$

To find the optimal average payoff, this paper solves for the partial derivative of the two maximum payoff

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equations with respect to the three θ values.

$$\frac{\partial \$_1}{\partial \theta_1}=0, \quad \frac{\partial \$_1}{\partial \theta_2}=0, \quad \frac{\partial \$_1}{\partial \theta_3}=0, \quad \frac{\partial \$_2}{\partial \theta_1}=0, \quad \frac{\partial \$_2}{\partial \theta_2}=0, \quad \frac{\partial \$_2}{\partial \theta_3}=0.$$

Factoring and employing cosine double-angle formulas and Pythagorean identities:

$$\frac{\partial \$_1}{\partial \theta_1}=2\cos\left(\frac{\theta_1}{2}\right)\sin\left(\frac{\theta_1}{2}\right)\left(-\cos^2\left(\frac{\theta_2}{2}\right)\cos^2\left(\frac{\theta_3}{2}\right)+\sin^2\left(\frac{\theta_2}{2}\right)\sin^2\left(\frac{\theta_3}{2}\right)\right)=0,$$

$$\frac{\partial \$_1}{\partial \theta_2}=-\cos\left(\frac{\theta_2}{2}\right)\sin\left(\frac{\theta_2}{2}\right)\cos(\theta_1)=0,$$

$$\frac{\partial \$_1}{\partial \theta_3}=\cos\left(\frac{\theta_3}{2}\right)\sin\left(\frac{\theta_3}{2}\right)\left(\cos^2\left(\frac{\theta_2}{2}\right)-\sin^2\left(\frac{\theta_2}{2}\right)\right.$$
$$\left.-2\cos^2\left(\frac{\theta_1}{2}\right)\cos^2\left(\frac{\theta_2}{2}\right)+2\sin^2\left(\frac{\theta_1}{2}\right)\sin^2\left(\frac{\theta_2}{2}\right)\right)=0.$$

$$\frac{\partial \$_2}{\partial \theta_1}=\cos\left(\frac{\theta_1}{2}\right)\sin\left(\frac{\theta_1}{2}\right)\cdot 0=0, \text{ therefore } \theta_1 \text{ can be any value.}$$

$$\frac{\partial \$_2}{\partial \theta_2}=-2\cos\left(\frac{\theta_2}{2}\right)\sin\left(\frac{\theta_2}{2}\right)\cos(\theta_3)=0,$$

$$\frac{\partial \$_2}{\partial \theta_3}=-2\cos\left(\frac{\theta_3}{2}\right)\sin\left(\frac{\theta_3}{2}\right)\cos(\theta_2)=0.$$

For $\frac{\partial \$_1}{\partial \theta_1}$ and $\frac{\partial \$_1}{\partial \theta_3}$ some parts can't be simplified further, although the theta values are the same. The two formulas can be combined into a system of equations. Since there are only two formulas with three variables, the system is underdetermined. To proceed, a simulation was used to identify the optimal combinations.

The resulting theta values are:

$\theta_1 \in \left\{0,\pi,\frac{\pi}{2}\right\}$ (taken from $\$_1$, since only player 1 can decide for round 1 (θ_1))

$\theta_2 \in \left\{0,\pi,\frac{\pi}{2}\right\}$ (taken from $\$_2$, for the same reason)

$\theta_3 \in \left\{0,\pi,\frac{\pi}{2}\right\}$ (taken from $\$_1$, for the same reason)

Instead of substituting each answer, a simulation with Qiskit was used to find which combination would give the optimal output.

2.2 Simulation

To simulate the algorithm, Google Colab was used with Python and Qiskit. Qiskit is an open-source software development kit, primarily built by IBM (IBM, 2017). It allows users to build, run, and optimize simple quantum circuits and algorithms on real quantum hardware and simulations.

Using Qiskit, this paper simulated the adopted algorithm using a three-qubit quantum circuit. Each qubit represented one round within the centipede game. Players' strategies were encoded using *Ry(-θ)* rotations, with $\theta \in \{0, \pi/2, \pi\}$, and the decisions were made (rotations were applied) while the three qubits were in a state of entanglement. Although the classical centipede game is played sequentially, the game model in Python applies the quantum strategies all at once, as shown in Figure 2. This is because quantum mechanics requires the entire system to evolve simultaneously and in a unified manner. Each round's strategy influences the quantum system as a whole, rather than being based on the outcomes of previous rounds, as would be the case in classical turn-based games. As discussed in previous works, such as Aerts (2004), quantum game models require that strategies be applied all at once due to the inherent nature of quantum evolution and measurement. Even if the decisions were applied one at a time, the resulting

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payoffs would have reflected the same results. After the strategies were applied, each qubit was disentangled and then measured to find the outcome of the game.

Using the equation derived in the previous section, the authors calculated the expected payoff for each round. For each combination of θ values, 1000 trials of the game were run to find the average payoff for each strategy applied. The resulting payoffs are displayed in Table 1.

For each of the 18 different θ configurations, 1000 trials of the quantum circuit were performed. In each trial, the state of all three qubits was measured and a bit string representing a classical state was obtained. This bit string was interpreted as a game outcome according to the following rule: if the first bit is 1, the outcome is considered a round-1 defection with payoff (1, 0); otherwise, if the second bit is 1, the outcome is considered a round-2 defection with payoff (0, 2); otherwise, the outcome is a round-3 with payoff (2, 2). Based on this information, the average payoff per strategy was calculated using the frequencies of outcomes, that is, $\$1 = p1 \cdot 1 + p3 \cdot 2$ and $\$2 = p2 \cdot 2 + p3 \cdot 2$, where $p1$, $p2$, and $p3$ correspond to the proportions of round 1, round 2, and round 3 outcomes, respectively. One important note, no round-3 defection was observed, verifying that $P_{3d1} = 0$. The obtained data are summarized in Table 1.

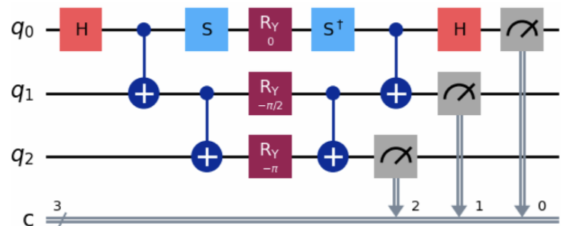

Figure 2: The quantum circuit above is the strategy circuit of a 3-round quantum centipede game. The q_0, q1, q_2 represent the 3 rounds of the centipede game. The H and CNOT gates create entanglement, the *S* gate adds the necessary phase difference, *Ry* boxes apply the *y* rotation, and the *c* line stores the results of the quantum measurements. Qiskit was used to visualize the circuit.

3. Results

Table 1 shows the final simulation result and the total payoff for each. The θ values [0, 0, 0] and [π, π, π] were found to have the most payoff per player, demonstrating two new Nash equilibria. These are the global maxima found after taking the partial derivatives of the average payoff equations. The other inputs tried were critical points resulting from partial differentiation. However, they are extraneous solutions because their average payoff is below 2 for either one or both of the players. Initially, only the combination of θ values (0, 0, 0) was thought to be the solution with the best payoff for both players, because the values indicate (classically) that players would cooperate in all three rounds. However, the findings contradict original assumptions. The simulation indicated that both (0, 0, 0) and (π, π, π) yielded the best outcome, which is to cooperate. How can there be 2 solutions when there is only 1 classical way to produce a payoff of (2,2)?

To verify this study's result, the authors substitute the θ values into (π, π, π), where $\theta_1 = \theta_2 = \theta_3 = \pi \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0, \beta_1 = \beta_2 = \beta_3 = 1$

Table 1. Raw averages of the payoffs for each strategy.

Strategy $[\theta_1, \theta_2, \theta_3]$	**Payoff** $[\$_1, \$_2]$
[0, 0, 0]	**[2.0, 2.0]**
[π, 0, 0]	[0.0, 2.0]
[π/2, 0, 0]	[0.96, 2.0]
[0, π, 0]	[1.0, 0.0]
[π, π, ,0]	[1.0, 0.0]
[π/2, π, 0]	[1.0, 0.0]
[0, π/2, 0]	[1.51, 1.02]
[π, π/2, 0]	[0.54, 0.92]
[π/2, π/2, 0]	[0.92, 0.96]
[0, 0, π]	[1.0, 0.0]
[π, 0, π]	[1.0, 0.0]
[π/2, 0, π]	[1.0, 0.0]
[0, π, π]	[0.0, 2.0]
[π, π, π]	**[2.0, 2.0]**
[π/2, π, π]	[1.08, 2.0]
[0, π/2, π]	[0.44, 1.12]
[π, π/2, π]	[1.56, 1.12]
[π/2, π/2, π]	[1.05, 1.14]

$$|\psi_{\text{unentangled}}\rangle = (0 + i \cdot 1)|000\rangle + (0 + i \cdot 0)|011\rangle + (0 + i \cdot 0)|101\rangle + (0 + i \cdot 0)|110\rangle = i\,|000\rangle$$

Similarly (0, 0, 0), where $\theta_1 = \theta_2 = \theta_3 = 0$

$$|\psi_{\text{unentangled}}\rangle = (1 + i \cdot 0)|000\rangle = |000\rangle$$

Although the player strategies differ, both sets of angles produce the same measurable state |000⟩, corresponding to "cooperate-cooperate-cooperate." The reason lies in the structure of the entangled wavefunction. The interference terms in the GHZ-type state cause the amplitudes associated with "defect in the final round" to cancel exactly, giving $P_{3di} = 0$. Once $P_{3di} = 0$, the branch of the wavefunction corresponding to last-round defection disappears entirely, leaving only the cooperative branch |000⟩, successfully avoiding the classical backwards induction trap.

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This leads to a remarkable symmetry in the strategy space. If all possible strategies were plotted as points in a three-dimensional cube with sides of length π, the two opposite corners, (0, 0, 0) and (π, π, π), are the Nash equilibria strategies and look like reflections of each other. Mathematically, when the strategies are substituted in the wave function, they only differ by a relative phase factor *i,* which cannot be observed when calculating probabilities since $(-1)^2 = 1^2 = 1$. The phase factor disappears and produces identical measurement probabilities. Intuitively, it is analogous to how two people walking towards a common point starting from opposite sides of a circle reach the same point. In summary, the player strategies can affect each other in the quantum centipede game in ways that are classically not possible, demonstrating that the quantum centipede game produces new cooperative Nash equilibria.

4. Discussion

Unlike classical game theory, which relies on fixed strategies and probability distributions, quantum game theory introduces a broader framework by incorporating principles of quantum mechanics like superposition and entanglement. Quantizing the centipede game allows for sustained cooperation. Entanglement correlates decisions across rounds, effectively binding the players' strategies together. Unlike classical reasoning, where each round is an isolated choice, this model creates a unified strategic state, preventing the backward induction logic that drives immediate defection.

To the author's knowledge, this is the first application of the continuous Eisert-Wilkens-Lewenstein protocol with maximal entanglement and full strategy space to a multi-round centipede game, as well as the first practical implementation of a quantum centipede game on Qiskit, an IBM back-end quantum simulation, to confirm findings. Expanding the set of possible outcomes and player strategies allowed entirely new approaches and types of equilibria that go beyond classical limits. This study's simulation revealed two coherent quantum Nash equilibrium profiles, both of which represent the classical path of cooperate-cooperate-cooperate. This outcome demonstrates a degeneracy: although the wave function for (π, π, π) differs from (0, 0, 0), both strategies' payoffs are equivalent.

Based on the findings of the analysis and the simulations carried out for the 3-round scenario, the following observation was made. Consider a sequential two-player, two-choice (cooperate/defect) game played over $n \geq 2$ rounds with players moving in order and round n being the last one. The authors assume the following for the game: there exists a classical SPNE using the backward induction method; there is quantization of the game as per the EWL approach; every round is a qubit represented as ($|0\rangle$= cooperate, $|1\rangle$ = defect); the players use local unitary transformations; rounds are entangled; the outcome is measured in the computational basis; and payoff is calculated from the final quantum state obtained. Given the above set of preconditions, then the following events may happen. First, backward induction fails to hold for the quantized game in question, where defection at round m occurs with probability zero. Secondly, there is an occurrence of symmetry due to entanglement, such that different classical states, say (0, 0, 0) and (π, π, π), lead to almost identical quantum states like $i|000\rangle$ and $|000\rangle$. As a consequence, similar cooperative payoffs are obtained. However, the origin of such symmetry from (0, 0, 0) to (π, π, π) (is this reflection in strategy space geometry?) still needs to be explored further. This paper proves that such a case occurs when n = 3. The authors believe that this could also apply to the cases where n > 3, and more research is encouraged to explore this topic more.

Lastly, this research also demonstrates how certain qubits may carry only phase information that does not affect the final measurement. In the 3-round centipede game case, since $P_{3di} = 0$, the last qubit is always measured as $|0\rangle$ (cooperation). Its associated unitary operator can therefore be taken as the identity ($U_3 = I$), meaning the last qubit no longer influences the calculation or payoff structure.

These strategies would not exist in a classical framework, yet they mirror choices frequently observed in experiments. The centipede game, in particular, has long demonstrated a disconnect between theoretical predictions and observed human behavior. Classical theory predicts immediate defection, while experimental studies show sustained cooperation. This study successfully shows how quantum algorithms can better model how the game plays out in real life. According to Healy (2024) studying the decisions of the human player, no players chose to defect in early rounds, while around 31 percent of players chose to split the pot at the end of the game. Another study (Dal Bó

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& Fréchette, 2011) shows that in the early rounds of the centipede game, cooperation rates were around 70-75 percent, reinforcing the idea that classical models alone fail to accurately capture human behavior. This paper's findings suggest that quantum game theory is a promising tool for bridging this gap.

5. Conclusion

In this research, the authors modified the EWL algorithm for a 3-round centipede game, and discovered two quantum Nash equilibria, at (0, 0, 0) and (π, π, π), both yielding a payoff of (2, 2), which is cooperation. Defection probability at the last round is found to be precisely zero. This finding eliminates the backward induction dilemma, and these results were confirmed using Qiskit simulation.

These contributions validate the feasibility of implementing quantum game protocols practically. They also open new pathways to applying quantum games to models of real-life interactions. Game theory has many different applications in many fields. Historically, game theory has been used by economists to study the effects of self-interest, rationality, and equilibrium in situations where traditional price theory does not apply. It models strategic interactions between different corporations, clients, and consumers by using simple games, such as those studied in this paper (Gibbons, 1997). Beyond economics, game theory is also used to model legal decisions and explain how people make choices that affect both themselves and others. However, these classical models often struggle to accommodate decisions that don't align closely with achieving maximal payoff. By introducing quantum strategies to solve these smaller games and create non-classical forms of equilibria, this work offers a better way to capture the nuances of human behavior and may serve as a contributor to next-generation tools in behavioral science, economics, and beyond.

This study has the following limitations: The centipede game is limited to a 3-round centipede game due to computational limits and complexity. The authors simulated the algorithm on an ideal, noiseless backend. For future research, noise on a suboptimal simulation could be introduced to see how it would affect the calculations of the algorithm. This goes hand-in-hand with simulating this paper's algorithm on real quantum hardware once the technology supports it. Real quantum computers would be affected by their surroundings, and things like noise, errors, and unstable qubits could change the results of the game. Testing the algorithm on real hardware would help show if it still works well outside of a perfect simulation.

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References

Aerts, D. (2004). Towards a quantum evolutionary scheme: Violating Bell's inequalities in language. arXiv preprint quant-ph/0407150. https://arxiv.org/pdf/quant-ph/0407150

Dal Bó, P., & Fréchette, G. R. (2011). The evolution of cooperation in infinitely repeated games: Experimental evidence. American Economic Review, 101 (1), 411-429. https://doi.org/10.1257/aer.101.1.411

Das, S. (2023). Quantumizing classical games: An introduction to quantum game theory. arXiv preprint 2305.00368v1. https://arxiv.org/pdf/2305. 00368v1

Eisert, J., Wilkens, M., & Lewenstein, M. (1999). Quantum games and quantum strategies. *Physical Review Letters*, 83(15), 3077-3080. https://doi.org/10.1103/physrevlett.83.3077

Elmshauser, B. (2022). Altruism and ambiguity in the centipede game. https://doi.org/10.31235/osf.io/93p8s

Feynman, R. P. (1982). Simulating physics with computers. *International Journal of Theoretical Physics*, 21 (6-7), 467-488. https://doi.org/10.1007/bf02650179

Forder, R. A. (1998). Military operations research: Quantitative decision making. *Journal of the Operational Research Society*, 49(11), 1227 – 1228. https://doi.org/10.1057/palgrave.jors.2600040

Frąckiewicz, P. (2017). A new quantum scheme for normal-form games. arXiv preprint 1701.07096. https://arxiv.org/pdf/1701.07096

Ghosh, I. (2021). Quantum game theory-I. *Resonance*, 26(5), 671-684. https://doi.org/10.1007/s12045-021-1168-2

Gibbons, R. (1997). An introduction to applicable game theory. *Journal of Economic Perspectives*, 11 (1), 127-149. https://doi.org/10.1257/jep.11.1.127

Healy, P. J. (2024). Epistemic experiments: Utilities, beliefs, and irrational play [Unpublished manuscript]. Department of Economics, The Ohio State University. https:/ /healy.econ. ohio-state.edu/papers/Healy-EpistemicExperiments.pdf

Holt, C. A., & Roth, A. E. (2004). The Nash equilibrium: A perspective. Proceedings of the National Academy of Sciences, 101 (12), 3999-4002. https://doi.org/10.1073/pnas.0308738101

Ibarrondo, R., Gatti, G., & Sanz, M. (2022). Quantum vs classical genetic algorithms: A numerical comparison shows faster convergence. In 2021 *IEEE Symposium Series on Computational Intelligence* (SSCI). https://doi.org/10.1109/ssci51031.2022.10022159

IBM. (2017). Introduction to Qiskit. IBM Quantum Documentation. https://quantum.cloud. ibm.com/docs/en/guides

Landsburg, S. E. (2011). Quantum game theory. In Wiley Encyclopedia of Operations Research and Management Science. https://doi.org/10.1002/9780470400531.eorms0697

Lemke, C. E., & Howson, J. T. (1964). Equilibrium points of bimatrix games. *Journal of the Society for Industrial and Applied Mathematics*, 12(2), 413-423. http://links.jstor.org/sici?sici=0368-4245%28196406%2912%3A2%3C413%3AEP0BG%3E2.0.C0%3B2-R

McKelvey, R. D., & Palfrey, T. R. (1992). An experimental study of the centipede game. *Econometrica*, 60(4), 803-836. https://doi.org/10.2307/2951567

Meyer, D. A. (1998). Quantum strategies. arXiv preprint quant-ph/9804010. https://arxiv.or g/pdf/quant-ph/9804010

Rao, V. R., Yang, N., & Zakerinia, S. (2024). Using quantum game theory to model competition. https://doi.org/10.2139/ssrn.5048345

Roughgarden, T. (2016). CS261: A second course in algorithms, Lecture #10: The minimax theorem and algorithms for linear programming. https://theory.stanford.edu/-tim/w16/1/110.pdf

Sekhon, R., & Bloom, R. (2020). Maximization by the simplex method. In Applied Finite Mathematics. Mathematics LibreTexts. https://math.libretexts.org/Bookshelves/Applied_Mathematics/Applied_Finite_Mathematics_(Sekhon_and_Bloom)/04%3A_Linear_Programmi ng_The_Simplex_Method/4.02%3A_Maximization_By_The_Simplex_Method

Simon, D. R. (1994). On the power of quantum computation. In Proceedings of the 35th Annual Symposium on Foundations of Computer Science (FOCS) (pp. 116-123). https://www.rese archgate.net/profile/Daniel-Simon-11/publication/2822536_0n_the_Power_of_Quantum_Computation/links/55ccbf3608aeca56cc1c185/0n-the-Power-of-Quantum-Computation.pdf

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ISSN 2688-3651 [online]

Styer, D. (1999). A brief history of quantum mechanics. https://www2.oberlin.edu/physics/dstyer/StrangeQM/history.html

von Neumann, J. (1928). Zur Theorie der Gesellschaftsspiele. *Mathematische Annalen*, 100(1), 295-320. https://doi.org/10.1007/bf01448847

Yildiz, M. (2012). Backward induction. In Economic Applications of Game Theory (Chapter 9). MIT OpenCourseWare. https://ocw.mit.edu/courses/14-12-economic-applications-of-game-theory-fall-2012/4b4412575dc74593c9d9c59e94427b69_MIT14_12F12_chapter9.Pdf

Author Contribution Statement

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